Sujet de Stage de M2R

Parameter estimation for stochastic differential equation: a bayes approach for quantum non-demolition measurement

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The purpose of this internship is to address problem of parameter estimation for particular stochastic differential equations. These equations model quantum measurements. more precisely they describe the evolution of a quantum system subject to some indirect quantum measurements. It is worth noticing that understanding the physical context is not required for this internship.

The involved equations are stochastic differential ones driven by Levy processes (simple models can be described by Wiener processes as well as Poisson processes). The law of the process is a mixture of Levy process laws. The goal of this internship is to understand the maximum likelihood for Levy processes and use it to analyze parameter estimation for the stochastic differential equation parameters. As a starting point, the interested student will study a discreet time version of the problem involving i.i.d. random variables instead of Levy processes. This part of the project consist in reading and understanding the results and proofs of [1]. In this article, it is shown that the maximum likelihood estimator is consistent and asymptotically mixed normal, and the statistical model is mixed local asymptotically normal. This implies that no estimator is more efficient than the maximum likelihood in that it saturates a Cramér–Rao bound.

It is required to have followed a course of stochastic calculus and asymptotic statistics at the level of M2 for this internship. The final goal of this internship is to establish results similar to [1] in the context of stochastic differential equations. This could end up with a first published research article which could be a good starting point for a PhD. This subject is typically at the intersection of probability, statistics and mathematical physics, and can be continued into PhD.

References

[1] *Quantum non demolition measurements: parameter estimation for mixtures of multinomials*, T. Benoist, F. Gamboa and C. Pellegrini, Electron. J. Stat. **12**(1) (2018) 555 – 571